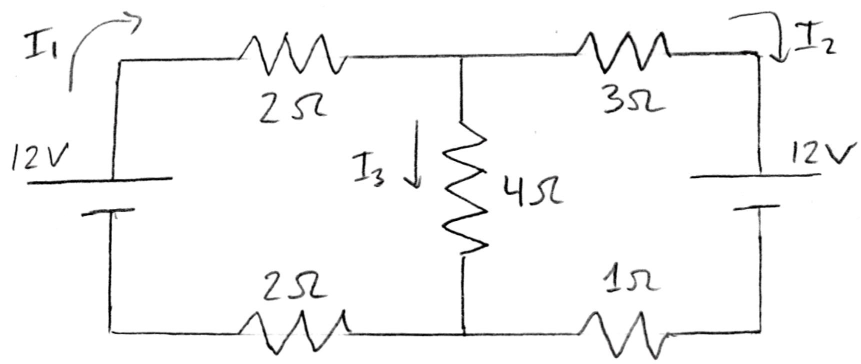
How to use Kirchhoff's Laws

to solve circuit problems with lots of resistors

With any math/physics/logic problem, the first thing I do is **name the unknowns**.

1. Currents are I_1 , I_2 , I_3 , etc. Voltages are *V*'s, resistances are *R*'s, etc.
2. For each current, draw an arrow pointing the direction you think current flows. If you guess wrong, that's OK; you'll just get a negative answer for that *I*.

Here is an example circuit with 2 batteries, 5 resistors, and unknown currents. (I left out capacitors and inductors because those things involve time-dependent currents. Except for some special cases, the full theory of time-dependent circuits requires solving differential equations.)



For each loop in the circuit, I use a conservation-of-energy method which I call **follow the money**. (It should really be called "follow the energy.") Here's how it works:

1. Pick a starting point somewhere in the loop. It doesn't matter where.
2. Imagine you are a (hypothetical) particle with 1 Coulomb of charge. Imagine the batteries and resistors in the loop are obstacles in your path.
3. A *V* Volt battery gives you $*V* if you go through it forwards. ("Forwards" means small plate, then big plate.) It costs you $*V* if you go backwards.
4. Going through an *R* Ohm resistor in the same direction as an arrow costs $*IR*.
   Going through the resistor in the other direction gives you $*IR.*

Keep going until you get back to the starting point. Add up all the costs and profits. **Kirchhoff's Voltage Law** says your total profit/loss should be zero for each loop.

For the example above, there are 3 loops: left side, right side, and all the way around the outside. I don't need to follow all 3 loops because 2 loops is enough to hit every obstacle at least once. For the left loop, I'll start at the top left corner. For the right loop, I'll start at the top right corner.

$$\text{Left loop} \qquad -2I_1 - 4I_3 - 2I_1 + 12 = 0$$
$$\text{Right loop} \qquad -12 - I_2 + 4I_3 - 3I_2 = 0$$

The next thing I do is **check all the junctions.** A *junction* is a point in the circuit where one current splits into two (or more) parts. **Kirchhoff's Current Law** says: the current going into each junction has to equal the current coming out of it. This law gives you more equations with *I*'s in them.

In my example, there are 2 junctions. Both of them require $I_1 = I_2 + I_3$. Together with the 2 loop equations, I now have 3 equations and 3 unknowns. The last step is to **solve a system of equations**. I write these systems in a standardized way:

1. Get each equation into the form $a\ I_1 + b\ I_2 + c\ I_3$ + etc = voltage.
2. Arrange the equations in rows and do matrix math.

$$\begin{aligned} 4I_1 + 4I_3 &= 12 \\ -4I_2 + 4I_3 &= 12 \\ I_1 - I_2 - I_3 &= 0 \end{aligned} \qquad \begin{bmatrix} 4 & 0 & 4 \\ 0 & -4 & 4 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 0 \end{bmatrix}$$

The system now looks like (matrix) * (column vector of *I*'s) = (column vector of *V*'s). In general, systems of equations are hard to solve. Writing them this way lets me use matrix tricks.

If you only have 2 variables and 2 equations, try this method:

1. Pick a "favorite" variable. Solve one equation for that variable.
2. Substitute your result into the other equation.

With more than 2 variables, things get complicated quickly. On an exam, you might want to skip to the next question and come back if you have time. If you get the physics right but can't solve the system of equations, you'll usually get substantial partial credit. More importantly: you've converted a physical problem into something which a computer can solve!

Here's one way to solve the example: start by subtracting row 2 from row 1 to find $I_1 = -I_2$. Substitute that into row 3 to find $2I_1 = I_3$. Use those results and row 2 to find $I_1 = 1$ and $I_2 = -1$. Last, use row 3 again to find I_3 . (There are many other ways to solve the same system.)

$$\begin{aligned} 4I_1 + 4I_2 = 0 \ &\Rightarrow\ I_1 = -I_2 \\ I_1 + I_1 - I_3 = 0 \ &\Rightarrow\ 2I_1 = I_3 \\ 4I_1 + 4(2I_1) = 12 \ &\Rightarrow\ I_1 = 1 \\ \Rightarrow\ I_2 = -1 \ &\Rightarrow\ I_3 = 2 \end{aligned}$$

Notice that $I_2 = -1$. This means I_2 is 1 Amp, but I drew the arrow in the wrong direction.

For big systems, or systems that change in time, engineers typically use computer programs designed by math nerds like me. (If you want to solve big systems yourself, check out some textbooks on vector spaces, linear algebra, and eigenvalue problems. For time-dependent systems, you'll need to learn ordinary differential equations, complex numbers, and Laplace transforms.)